University of California, Santa Cruz
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Electrical Engineering Department

Laboratory 5
Sinusoidal Steady State Filters

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EE101L
Intro. to Electronic Circuits Laboratory

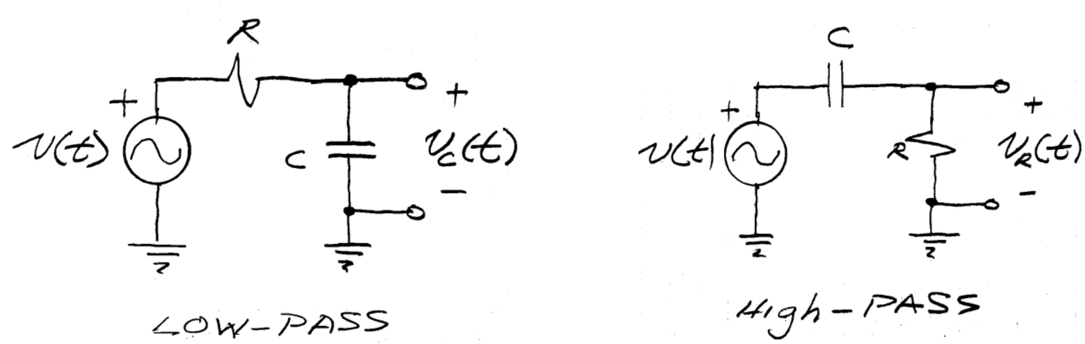
1. DESCRIPTION AND OBJECTIVE

This laboratory investigates the frequency domain characteristics of steady state linear wave filters. Working in the phasor domain, we will investigate salient characteristics of the basic first order high-pass and low-pass filters, and second order bandpass and bandreject filters. Amplitude and phase responses will be experimentally measured and correlated with underlying theory. A first order low-pass filter using an operational amplifier will also be constructed. This lab affords a better understanding of bode plots, decibels and decibel notation, as well as phase and its fundamental relationship to periodic time.

2. GENERAL DISCUSSION

First Order Series RC Filters

A series connection of a resistor and a single reactive element, capacitor or inductor, can be arranged to implement a low or high-pass filter. These are shown below for the capacitor.:



When driven by a steady-state sinusoidal voltage, the response will exhibit two characteristics. First, the steady-state AC output voltage will vary with frequency; this is generally specified as $20\log\frac{|v_{out}|}{|v_{in}|}$ [dB] versus $\log$ frequency, where both the input and output must be identically expressed (this could be rms, peak or peak-to-peak). The log-log X vs.Y plotting technique allows us to see the response as a function of frequency as piece-wise continuous straight line segments known as a *bode plot*. Second, the time delay will vary with frequency. Expressing the response in decibles provides a convenient way to view the amplitude variation over a potentially wide range, typically many orders of magnitude.

Second, the response sine wave will shifted in phase with respect to the input sine wave. This is measured first as a time delay and then expressed as a leading or lagging phase shift with respect to the input phasor.

Low-pass circuit.

Consider first the low-pass RC circuit. Using phasors and the voltage divider theorem, we can write the low-pass response voltage across the capacitor with the input being the reference phasor as

[1] $$\overline{V}_{out} = \overline{V}_{in}\frac{Z_C}{R+Z_C} = V_{in}\angle 0^\circ \frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}} = V_{in}\angle 0^\circ \frac{1}{1+j\omega RC}$$

This equation tells us that as frequency increases, the response will monotonically decrease. How do we characterize this? At low frequencies, $\overline{V}_{out} \approx \overline{V}_{in}$ and at zero hertz they are identical in amplitude, but as frequency approaches infinity, the response goes to zero. By convention, we agree to define the "breakpoint", "cutoff" or "corner" frequency, f_c, at the half power or -3dB point. All frequencies to the left of f_c are considered those we want and is appropriately called the *passband*. Those to the right are considered those we don't want and is accordingly called the *stop-band*. The "roll-off rate" beyond the cutoff frequency in the stop-band for any RC filter is -6dB per octave (twice frequency) or -20dB per decade (10 times frequency). Remember that a change in voltage of 2 corresponds to +6dB, or $20\log 2$, so -6dB corresponds to a change of one half, or $20\log\frac{1}{2}$. Note the argument of the logarithm is always a dimensionless number. Here it is the ratio of output over input voltage as noted earlier: $20\log\frac{|v_{out}|}{|v_{in}|}$. Hence if we double the frequency in the stop-band, the amplitude will drop in half, and if we increase it by a factor of 10, it will be attenuated by 10. The bode plot shows this as a straight-line having a negative slope in dB per log frequency.

At the corner or break frequency, the resistance and reactance have the same magnitude: $R = |Z_C|$. Hence,

[2] $$\omega_c RC = 1 \Rightarrow \omega_c = \frac{1}{RC} \Rightarrow 2\pi f_c = \frac{1}{RC} \Rightarrow \boxed{f_c = \frac{1}{2\pi RC}}$$

We can also derive the output voltage as a function of the input voltage at f_c.

[2] Thus, after some algebra, $\overline{V}_{out} = V_{in}\frac{1}{\sqrt{2}}\angle -45^\circ$. Converting this result back to the time-domain gives

[3] $$v_{out}(t) = \mathrm{Re}\left\{V_{in}\frac{1}{\sqrt{2}}e^{j\omega t}e^{-j45^\circ}\right\} = \mathrm{Re}\left\{V_{in}\frac{1}{\sqrt{2}}e^{j(\omega t-45^\circ)}\right\}$$
$$= \frac{V_{in}}{\sqrt{2}}\mathrm{Re}\left\{\cos(\omega t-45^\circ) + j\sin(\omega t-45^\circ)\right\} = \frac{V_{in}}{\sqrt{2}}\cos(\omega t-45^\circ)$$

The output voltage lags the input voltage by 45 degrees at the cutoff frequency.

High-pass circuit.

Interchanging the resistor and capacitor results in a high-pass filter. As you might expect it has the same cutoff frequency as the low-pass does, but passes all frequencies above f_c rather than attenuating them. Again, using phasors and the voltage divider theorem, we can write the high-pass response voltage across the resistor as

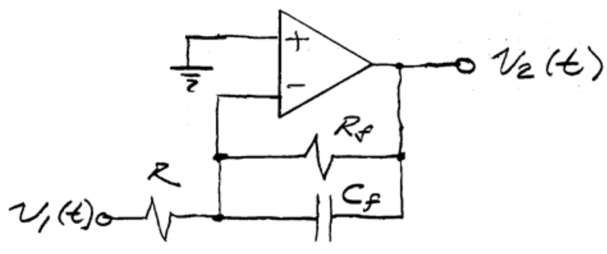
[4] $$\overline{V}_{out} = \overline{V}_{in}\frac{R}{R+Z_c} = V_{in}\angle 0^\circ \frac{R}{R+\frac{1}{j\omega C}} = V_{in}\angle 0^\circ \frac{j\omega RC}{1+j\omega RC}$$, and at f_c

[5] $\overline{V}_{out} = V_{in}\frac{1}{\sqrt{2}}\angle 45^\circ$. Here, the output voltage leads the input voltage by 45 degrees.

First order series RL filters can also be used to implement high and low-pass filters. In this lab we will only investigate the RC variety.

Operational amplifier 1st order low-pass filter circuit

Op-amps are frequently used to implement wave filters. The circuit below shows a single op-amp configured as a first order low-pass filter. Since this is not a passive circuit, we can design it to amplify the input signal as well as filter it. The general circuit is shown below.



The response when the input is a steady-state sinusoid is given by

[6] $\overline{V}_0 = \overline{V}_{in}\left(-\frac{Z_f}{R}\right)$, where the feedback impedance Z_f is the parallel combination of R_f and C_f ,

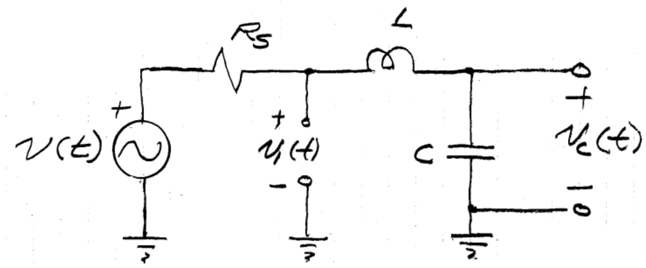
$$\overline{V}_0 = \overline{V}_{in}\left(-\frac{R_f}{R}\frac{1}{(1+j\omega R_f C)}\right)$$

The cutoff frequency is similar to the passive circuit's, $f_c = \frac{1}{2\pi R_f C_f}$.

2nd Order RLC filters

Series and parallel connections of RLC can be used to implement additional filter types beyond the basic high and low-pass filters. In this lab we will again use the same circuits and components used in Lab-4, but drive them with steady-state sinusoids intead of stepped voltages.

Series RLC Bandpass and Bandreject filters

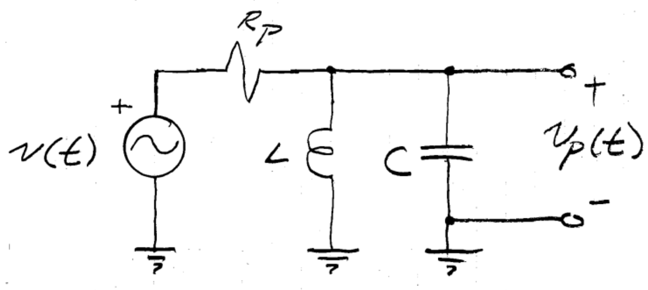


When the response is taken across the capacitor, the circuit above exhibits a *bandpass* response. $v_C(t)$ will be maximum only at (or close to) the natural resonant frequency of L and C, namely at $f_0 = \frac{1}{2\pi\sqrt{LC}}$. At this particular frequency $X_L = X_C$. This is not a coincidence. As you saw in the last lab, L and C left to themselves will naturally ring at this frequency. Only when this circuit is driven with $v(t) = V\cos(2\pi f_0 t)$ will

[7] $Z_3 = j\omega L + \frac{1}{j\omega C} = jX_L - jX_C = 0$ Ohms looking into the series LC branch at the node where $v_1(t)$ is connected. If the output is taken across the series LC branch, $v_1(t)$, it will exhibit a *bandreject* response. At f_0 theoretically (ideally) there will be no output since the series combination of L and C will look like a short-circuit, leaving only the series resistance, R_S, to limit the current. Moreover, i_S will now be in phase with v_1. Eqn. (7) clearly shows that above this frequency, $X_L > X_C$ and Z_3 will look inductive. Below, $X_L < X_C$ and it will look capacitive. Further, since Z_3 is zero when $X_L = X_C$, the series current will be largest at f_0. These facts should be clear from the equation for the series current,

[8] $\bar{I}_S = \frac{V\angle 0^\circ}{R + j\omega L + \frac{1}{j\omega C}} = \frac{V\angle 0^\circ}{R + jX}$.

Parallel RLC Bandpass filter



The parallel resonant LC circuit has an interesting property. When L and C are ideal, at their natural resonant frequency, $f_0 = \frac{1}{2\pi\sqrt{LC}}$, they form an equivalent open circuit, $Z_p = \infty$. Above and below this frequency they look capacitive, $-jX_C$ and inductive, $+jX_L$ respectively. Hence, this forms a voltage divider:

[9] $\overline{V}_P = V\angle 0^\circ \frac{Z_p}{Z_p + R}$.

So, at f_0, as $Z_p \to \infty$ $\overline{V}_P \to V\angle 0^\circ$. At this point $\overline{V}_P$ is maximum and we are at the center of the bandpass filter's response. Notice that as the frequency goes to zero or infinity, $\overline{V}_P$ also goes to zero.

3. Experimental work

For each of the circuits investigated below, begin with an accurately drawn engineering schematic of your *actual experimental* circuit that includes the signal generator's output resistance and the parasitic loading of the scope probe. As in the previous lab, be sure to set the scope probe to "10x" not "1x", as the 10x setting will decrease loading effects of the probe (this is printed on the body of the scope probe where it plugs into the oscilloscope); *note*: you must also tell the scope which setting you're using by setting the display to read "10X".

Drive each circuit with a sine wave and a voltage sufficient to get good scope displays.

3.1 Series RC low-pass filter

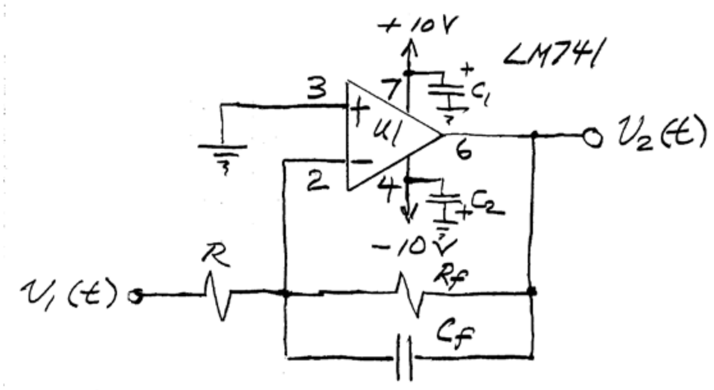
Construct an RC low-pass filter using a 100 nF capacitor and a resistor between 1k and 22k Ohms. Determine the corner frequency and the phase-shift at this frequency. Continue to talk enough measurements at other frequencies sufficient to construct bode and phase plots. Do this again for for a second value of $R_S = 50$ Obms (use the signal generator output resistance alone).

3.2 Series RC high-pass filter

Repeat what you did for the low-pass filter using the same components, but swapping the positions of the capacitor and resistor to take the output across the resistor. This will implement a high-pass filter. Do this for two values of R_P, 470 (in series with sig. gen. output resistance) and 4.7k.

3.3 Operational amplifier low-pass filter

Construct the experimental circuit shown using the LM741 op-amp in your lab kit. Be sure to bypass the positive and negative supply rails near the chip with capacitors at least 1 uF for each rail. Because the op-amp itself has a low-pass response, we must keep the low-pass corner frequency, $f_C = \frac{1}{2\pi R_f C_f}$ well below the op-amps -3dB corner frequency. To insure this is the case, use $C_f = 100$ [nF], and keep R_f between 22 [k] and 51[k]. Having chosen R_f, keep the closed-loop low-frequency gain, $G = -\frac{R_f}{R}$ [V/V] (when the capacitor has little or no effect) no greater than 24 [dB].



Characterize the filter, finding its' cutoff frequency and phase shift and taking enough data to construct accurate bode and phase plots. In your report, use phasor domain analysis to determine the op-amps gain and output.

3.4 Series RLC bandpass and bandreject filters

Construct the series RLC circuit using the same inductor and capacitor you used in Lab-4. Using both channels on the oscilloscope, view $v_1(t)$ on one channel and $v_C(t)$ on the other. Vary the sinusoidal frequency from the signal generator to find series resonance. This will occur when $v_1(t)$ is *minimum* and $v_C(t)$ is *maximum*. Theoretically verify that the phase shift across the capacitor will be 90 degrees with respect to $v_1(t)$ at this frequency and confirm it experimentally. Take enough points to plot response amplitude responses for both $v_1(t)$ and $v_C(t)$. $v_1(t)$ will exhibit a bandreject response, while $v_C(t)$ will exhibit a bandpass response. Determine the bandwidth of the bandpass filter.

3.5 Parallel RLC bandpass filters

Rewire the series circuit to implement the parallel RLC circuit shown using the same values for L and C. Use the dual-trace scope to view $v_1(t)$ on one channel and $v_C(t)$ on the second and verify basic properties of this filter. Show that at resonance, the two voltages are in phase and maximum and that the phase shift properly follows theory for frequencies on either side of resonance. Determine the bandpass filter's bandwidth for two different values of R and take enough data to plot and compare the two. Theoretically discuss and justify your results.

4. REPORT AND SUBMISSION.

Submit a report (see the handout on reporting guidelines; this is posted on our website) discussing the work done in this laboratory. Note that Bode plots require log-log graphs. Either plot these by hand or find a good graphing program. Do not use Excel.

For each filter type, derive the theoretical responses using phasors to verify your observations.