University of California, Santa Cruz
Baskin Engineering School
Electrical Engineering Department

Laboratory 4
Second Order Switched RLC Circuits

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EE101L
Intro. to Electronic Circuits Laboratory

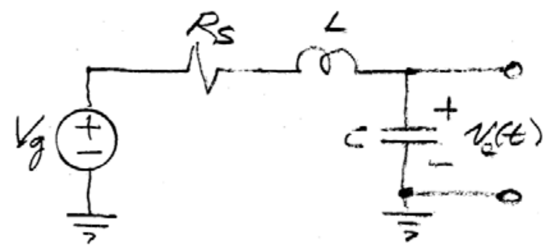
1. DESCRIPTION AND OBJECTIVE

This laboratory investigates transient voltage and current relationships for switched second order RLC circuits. Such circuits when excited by step voltages exhibit responses that will be experimentally observed and analyzed to verify circuit theory.

2. GENERAL DISCUSSION

Series RLC

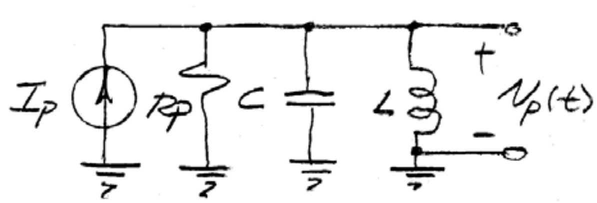
A series connected resistor, capacitor and inductor when excited by a step change in voltage will exhibit a pure sinusoidal oscillation when R_S is zero.



Increasing R_S causes power losses progressively attenuating or "dampening" these oscillations with time. When R_S reaches a critical value, where the losses are quite large and oscillations just begin to cease entirely, this is known as *critically damped*. Beyond this value, the network is considered *overdamped* and it behaves much like a single time constant network, but now having a decay that is the sum of two exponentials corresponding to the roots of the network's second order differential equation.

Parallel RLC

A parallel connected resistor, capacitor and inductor when excited by a step change in current will also exhibit a pure sinusoidal oscillation when R_P is infinite.



Decreasing R_P will cause losses that also progressively attenuate or dampen these oscillations exponentially with time. As with the series RLC circuit, when R_P decreases to the point of *critical damping,* the sinusoid is now damped faster than it can oscillate and oscillations cease. When decreased further, the network is considered *overdamped*. Notice that if we drove the parallel RLC with an ideal voltage source its' internal Thevenin resistance would short R_P causing no response whatever! Hence, at least theoretically, we always show this circuit driven with a current source.

General Step Response Equations

The complete general solutions for RLC circuits when driven by a voltage or current step, valid for either the series or parallel forms, is given by $y(t) = y_{ss}(t) + y_t(t)$. Variable $y(t)$ represents either the voltage or current as appropriate, and $y_{ss}(t) = y(\infty)$ the forced or steady-state final value response. The remaining term is the transient or natural response that necessarily dies out over time. The complete solutions for the three cases are summared below (see eqns. 8.44, text).

$s = \frac{-\alpha \pm \sqrt{\alpha^2 - \omega_0^2}}{2}$, found from the characteristic equation: $s^2 + 2\alpha s + \omega_0^2 = 0$

$\alpha = \frac{R_S}{2L}$, for the series RLC circuit, and $\alpha = \frac{1}{2R_P C}$, for the parallel RLC circuit.

α accounts for energy lost in either resistance, R_P or R_S , and is appropriately called the *damping factor*, since this energy is not recoverable.

$\omega_o = \frac{1}{\sqrt{LC}}$, the natural *undamped resonate frequency* is the same for both series and parallel circuits.

The solution forms for each case are:

$\alpha^2 - \omega_0^2 > 0$, Overdamped: $y(t) = y(\infty) + K_1 e^{s_1 t} + K_2 e^{s_2 t}$

$\alpha^2 - \omega_0^2 = 0$, Critically damped: $y(t) = y(\infty) + e^{-\alpha t}\left(K_1 + K_2 t\right)$

$\alpha^2 - \omega_0^2 < 0$, Underdamped: $y(t) = y(\infty) + e^{-\alpha t}\left(K_1 \cos(\omega_d t) + K_2 \sin(\omega_d t)\right)$

Where $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$, damped resonant frequency.

Constants are found from initial conditions for $y(0)$ and $\frac{dy(0)}{dt}$.

Note that as α increases, $\omega_d = 2\pi f_d$ decreases, so the period of Herzian oscillation, $T_d = \frac{1}{f_d}$ *increases*.

3. Experimental work

For each of the two circuits described below, begin with an accurately drawn engineering schematic of your *actual experimental* circuit that includes the signal generator's output resistance and the parasitic loading of the scope probe. Be sure to set the scope probe to "10x" not "1x", as the 10x setting will decrease loading effects of the probe (this is printed on the body of the scope probe where it plugs into the oscilloscope).

Drive each circuit with a square wave and a voltage sufficient to get good scope displays.

3.1 Step Response of a Series RLC Circuit

Investigate the three regions of operation for a series RLC circuit consisting of a 10 $[\mu H]$ inductor and 1000 $[pF]$ capacitor and selected resistors. Verify that as α increases, the damped resonant frequency f_d decreases and it takes fewer cycles to reach steady state. Experimentally determine the value of R_s needed for critical damping, Try to quantitatively estimate α , f_d and f_0 . Wire your circuit so you can view the voltage across the capacitor, not the inductor, since the scope probe has less loading effects on the capacitor than the inductor.

Since the signal generator has a 50Ω source resistance, the smallest that R_s can be is this value. So this is the value you should first begin with. Proceed to experimentally increase α by inserting different resistors from the lab's fixed resistor stock to observe, *over*, *under* and *critically damped* responses. For each case, observe the approximate number of cycles until steady state is reached and note f_d. Discuss this procedure in your lab report.

3.2 Step Response of a Parallel RLC Circuit

Rewire your capacitor and inductor so they are now in parallel and repeat the experiment, again noting that as α increases f_d decreases and it takes fewer cycles to reach steady state. Carefully consider the effect of the signal generator's internal source resistance, since now α varies inversely with R_p instead of directly with R_s as it did with the series RLC circuit. Again vary α by changing the resistance in series with the signal generator and try to quantitatively estimate α, f_d and f_0.

4. REPORT AND SUBMISSION.

Submit a report (see the handout on reporting guidelines; this is posted on our website) discussing the work done in this laboratory. From your work compare and contrast the series and parallel circuits. Because of these differences, you should have been able to decrease α for the parallel circuit below that otherwise attainable from the series circuit. Discuss why this is and what the consequences were experimentally. In particular, explain how you were able to drive the parallel circuit with a voltage source and not have it kill the underdamped oscillations.

Include in your report full derivations of the general 2^{nd} order differential equations for both the series and parallel cases, then based on particular experimental values (for R, L, C) how the form of the appropriate particular solution applies.