University of California, Santa Cruz
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Laboratory 2
Fundamental Circuit Theory Theorems

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EE101L
Intro. to Electronic Circuits Laboratory

1. DESCRIPTION AND OBJECTIVE

Three theorems fundamental to the study of circuit theory are explored in this laboratory. The first two are classic engineering models designed to represent lumped linear non-ideal voltage and current sources. These are known as the *Thevenin* equivalent circuit for the non-ideal voltage source, and *Norton* equivalent circuit for the non-ideal current source. We will then use these engineering models to consider what optimum load resistance is required for maximum power to flow from a non-ideal source into a resistive load that experimentally confirms the *maximum power transfer theorem*.

2. GENERAL DISCUSSION

Thevenin and Norton Engineering Models:

The basic toolbox needed for lumped circuit analysis includes only *ideal* linear components and elements. These include: passive resistors, capacitors, inductors, voltage and current sources. Non-ideal lumped circuits can then be designed with suitably constructed engineering models composed only of these basic ideal linear blocks[1]. Probably the most fundamental of these are the Thevenin and Norton equivalent circuits shown in fig. 1.

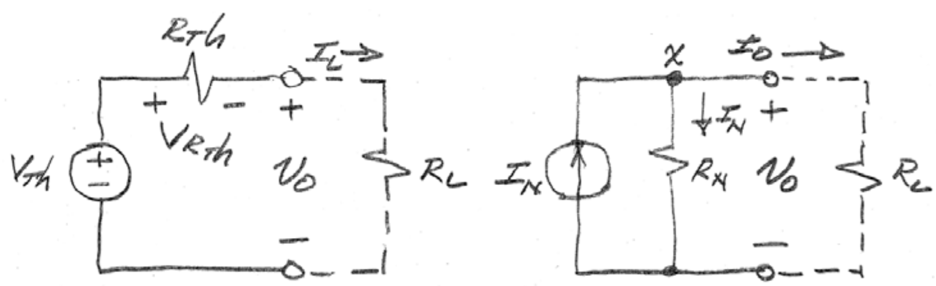

Fig. 1. Fundamental Thevenin and Norton equivalent circuits.

The Thevenin model accounts for the fact that a voltage source's output voltage, V_O, varies with supplied current by adding a series resistor, R_{th} to satisfy KVL around the single mesh: $V_{Th} = V_{Rth} + V_0$. The Norton model accounts for the fact that a current source's output current, I_O, varies with supplied voltage by adding a shunt resistor to satisfy KCL at node *X*: $V_N = I_N + I_O$. These models are frequently used in circuit theory to simplify or "reduce" otherwise complicated networks into either of these two equivalent circuits. This may involve all or just portions of

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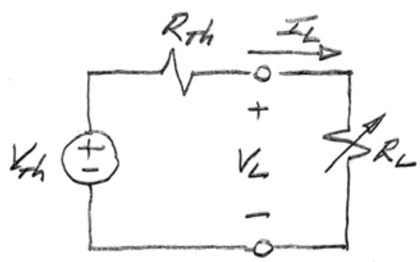
[1] Any circuit containing only linear devices and elements is itself also linear. This will be useful later when we use this fact to apply the superposition theorem.

such networks. Moreover, since it can be proven that the two models are electrically equivalent with respect to whatever is connected as a load, R_L , either model can easily be converted into the other by *source transformation*.

Maximum Power Transfer

Power flowing from an *ideal* source to a resistive load always transfers all of its generated power into the load itself, i.e. $P_{source} = P_{load}$. The actual power of course is set by the value of the load resistance, but the generator will always supply all of it. Practical non-ideal sources, however, have internal source resistances that inevitably dissipate some of the generated power that would otherwise flow into the load, *i.e*, $P_{source} - P_{loss} = P_{load}$. Since the power division between the load and internal loss is determined solely by the source and load resistances, the practical question arises regarding what these need to be to realize maximum power flowing into the load. An audio amplifier, for example, driving a 4 Ohm or 8 Ohm speaker must consider this question. The answer is given by a famous fundamental theorem of electic circuits. The *maximum power transfer theorem* tells us how to *match* source and load resistances. Later you will see this theorem holds for AC circuits as well, where resistanes are generalized as impedances.

The maximum power transfer theorem states that the largest amount of power that can be coupled from a non-ideal source into a load occurs when the load resistance is equal to the source resistance[2]. To help you see this, consider the following non-ideal voltage source with a fixed and unchangeable source resistance, R_S , connected to a resistive load, R_L :



Now find R_L such that maximum power is transferred to the load from the source. The power in the load is given by $P_L = I_L V_L$. Noting that

$V_L = V_{Th} \dfrac{R_L}{R_{Th} + R_L}$, by the voltage divider theorem and

$I_L = \dfrac{V_{Th}}{R_{Th} + R_L}$, by Ohm's law applied to the loop. We can the write the power in the load as

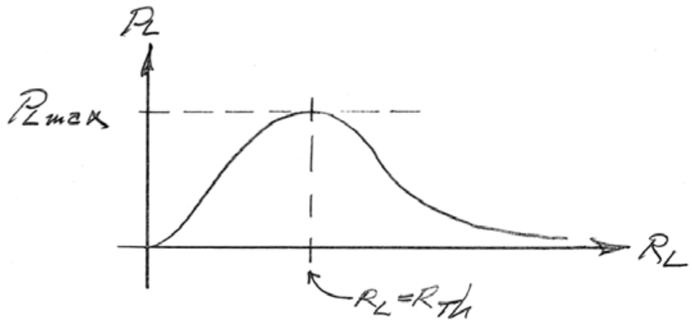
$P_L = \left(\dfrac{V_{Th}}{R_{Th} + R_L}\right) V_{Th} \dfrac{R_L}{R_{Th} + R_L} = \dfrac{V_{Th}{}^2 R_L}{\left(R_{Th} + R_L\right)^2} = \dfrac{V_{Th}{}^2 R_L}{\left(R_{Th}\right)^2}$, where R_T is the sum of source and load resistances.

Differentiating this equation and setting it to zero gives us the value of R_L we are looking for.

$\therefore \dfrac{dP_L}{dR_L} = \dfrac{R_T{}^2 V_{Th}{}^2 - V_{Th}{}^2 R_L 2R_T}{R_T{}^4} = 0$. Solving yields $R_T = R_L + R_{Th} = 2R_L$, $\Rightarrow \boxed{R_S = R_L}$

[2] This derivation is given in sec 4.8 of our textbook.

Plotting P_L as a function of R_L clearly reveals that when the load resistance equals the source resistance maximum power flows from the source into the load:



Thus, $P_{L\max} = \dfrac{V_{Th}{}^2 R_{Th}}{\left(R_{Th} + R_{Th}\right)^2} = \dfrac{V_{Th}{}^2 R_{Th}}{4R_{Th}{}^2}$. Therefore, $\boxed{P_{L\max} = \dfrac{V_{Th}{}^2}{4R_{Th}}}$

We can understand the significance of the somewhat strange cofactor $\dfrac{1}{4}$ by appreciating that conceptually $\dfrac{V_{Th}{}^2}{R_{Th}}$ is *not* the total power available, but the power dissipated in the internal Thevenin resistance *if the load resistance* $R_L = 0$. When the load and source are equal, only half of this is now available and this is equally divided between R_L and R_{Th}.

A note on perspective is in order here. There are many cases where what we are interested in is *maximum voltage* across the load, or *maximum current* flowing through it, but not necessarily maximum power delivered to it. As already noted, and quite evident from the of P_L vs. R_L graph, maximum power occurs when $\dfrac{R_L}{R_{Th}}$ is equal to 1; this is when the volt-amp *product* is largest, but not necessarily the voltage or current alone. It should further be evident that maximum voltage occurs when this ratio is much greater then 1 and maximum current when it is much less than one. Examples are voltage amplifiers and current amplifiers. Power amplifiers require matching while voltage and current amplifiers do not.

3. Experimental work

3.1 Thevenin and Norton Equivalent Circuits

Reconstruct the same 2-port T-network that you used in the first lab and devise an experiment to determine the Thevenin and Norton equivalent circuits when driven at the input port by an ideal 10 V DC power source. Perspective is important here. Treat the power source and T-network as if they were combined together inside a mystery "black box" having only one port, an output port. Thus you can experimentally work *only* with the two nodes at the output port, 1 and 2, and whatever is connected to it. Fig. 2 shows the basic idea.

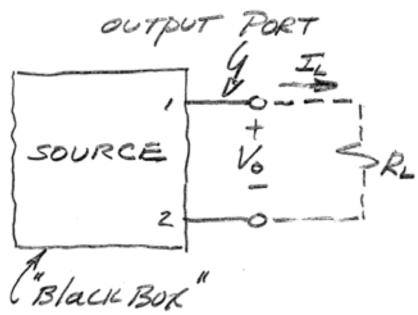


Fig. 2. Basic "black-box" 1-port source network.

Since the source transformation theorem relies on the fundamental equivalence of the two models, you should be able to deduce either model solely from your data. After you have experimentally determined both circuits, add a variable load, or use data already taken (with different discrete values for R_L), to analystically show that both engineering models can indeed be used to accurately predict any load current and voltage, I_L and V_L within the range permitted by the source.

Now, consider the practical question of just how ideal the lab supply connected to your T-network really is. Assess this by proceeding to experimentally determine the Thevenin source resistance of the experiment's power supply *at the point it is connected to your plugboard*. Just how ideal is it, and generally, does it need to be taken into account?

3.2 Maximum Power Transfer

Using either the Thevenion or Norton moldels you experimentally obtained for sec. 3.1, prove the maximum power transfer theorem experimentally.

4. REPORT AND SUBMISSION.

Submit a report (see the handout on reporting guidelines; this is posted on our website) discussing the work done in this laboratory.